1. Consider the market for good X. The market demand for X is given by

\[(1) \quad X_D = X_D(P)\]

where \(X_D\) represents the quantity of X demanded and \(P\) represents the market price of X. Assume \(X_D'(P)<0\) and \(X_D''(P)=0\).

The suppliers of good X must pay a tax on each unit of output they sell. Let \(t>0\) represent this per-unit tax. Let \(\rho=P-t\) represent the net, after-tax price received by the suppliers of good X. Assume \(P>t\). The market supply of X is given by

\[(2) \quad X_S = X_S(\rho)\]

where \(X_S\) represents the quantity of X supplied. Assume \(X_S'(\rho)>0\) and \(X_S''(\rho)=0\). Note that the suppliers base their output supply decisions on the net, after-tax price of X.

In equilibrium,

\[(3) \quad X_D(P) = X_S(\rho)\]

and

\[(4) \quad \rho = P - t.\]

a. Derive and sign expressions for the following comparative statics results. (NOTE: These expressions should consist of \(X_D'(P)\) and \(X_S'(\rho)\) terms only.) (15 points)

\[
\frac{dP}{dt} ; \frac{d\rho}{dt} ; \frac{dX}{dt}
\]

b. Present a completely labeled, “rough sketch” diagram to illustrate these comparative statics results for an increase in \(t\). Briefly discuss the economic intuition underlying your diagram. Be sure to address how your graphical analysis relates to your mathematical analysis. (10 points)

THIS SECTION CONTINUES ON THE NEXT PAGE.
c. How would you interpret a finding that \( \frac{dP}{dt} \) is “close to 1” and \( \frac{d\rho}{dt} \) is “close to 0”? How would you interpret a finding that \( \frac{dP}{dt} \) is “close to 0” and \( \frac{d\rho}{dt} \) is “close to –1”? (10 points)

d. Convert the comparative statics results in part a to “elasticity form.” Show all of your work. (NOTE: There is an “easy way” and a “hard way” to do this. Either way is acceptable. The easy way exploits the assumption that \( X_S \) is linear in \( \rho \) and the assumption that \( t \) is a per-unit tax on output. Think about a linear supply function where \( \rho \) is the independent variable. Then substitute \( \rho = P - t \) into this function. Think about the derivatives of these two functions.) (25 points)

e. Evaluate the limit of \( \frac{dP}{dt} \) and the limit of \( \frac{d\rho}{dt} \) as the elasticity of the demand for good X approaches 0. Interpret these results. Illustrate with a large, completely labeled diagram. (20 points)

f. Evaluate the limit of \( \frac{dP}{dt} \) and the limit of \( \frac{d\rho}{dt} \) as the elasticity of the supply of good X approaches 0. Interpret these results. Illustrate with a large, completely labeled diagram. (20 points)
2. The market for beer in Mt. Vermin is described by the following demand and supply functions

\[ B_D = B_D(P_B, P_Z, N(A)) \]
\[ B_S = B_S(P_B) \]

where \( B_D \) is the number of 6-packs of beer demanded, \( B_S \) is the number of 6-packs of beer supplied, \( P_B \) is the price of a 6-pack of beer, \( P_Z \) is the price of a bag of pretzels, \( N \) is the number of consumers of beer in Mt. Vermin, and \( A \) is the legal drinking age in Mt. Vermin. Note that \( N \) is a function of \( A \). Assume \( N'(A)<0 \). In addition, assume \( \frac{\partial B_D}{\partial P_B} < 0 \) and \( \frac{\partial B_S}{\partial P_B} > 0 \).

The market for pretzels in Mt. Vermin is described by the following demand and supply functions

\[ Z_D = Z_D(P_Z, P_B) \]
\[ Z_S = Z_S(P_Z, P_C) \]

where \( Z_D \) is the number of bags of pretzels demanded, \( Z_S \) is the number of bags of pretzels supplied, and \( P_C \) is the price of a bag of potato chips. Assume the firms that produce pretzels also produce potato chips. In addition, assume \( \frac{\partial Z_D}{\partial P_B} < 0 \) and \( \frac{\partial Z_S}{\partial P_B} > 0 \).

a. Assign signs to the following partial derivatives. Briefly justify each of these signs. (10 points)

\[ \frac{\partial B_D}{\partial A} ; \frac{\partial Z_S}{\partial P_C} \]

b. Derive and sign the following comparative statics results. Be sure to specify any additional assumptions you employ in your analysis. (30 points)

\[ \frac{dP_B}{dA} ; \frac{dB}{dA} ; \frac{dP_B}{dP_C} ; \frac{dB}{dP_C} \]
\[ \frac{dP_Z}{dA} ; \frac{dZ}{dA} ; \frac{dP_Z}{dP_C} ; \frac{dZ}{dP_C} \]

c. According to your analysis in part b, how will an increase in the legal drinking age affect \( P_B \), \( B \), \( P_Z \), and \( Z \)? Explain why these results make sense. Illustrate your explanation with completely labeled, “rough sketch” demand and supply diagrams. (15 points)
3. Answer all of the questions in this section within the context of the ISLM model we developed in class. You do not have to derive the comparative statics results we considered in class \([dy/dG), (dr/dG), (dy/dM_S)\) and \((dr/dM_S)\) “from scratch.” If you need to employ these comparative statics results, then simply state them from your class notes and use them.

a. Is there a “balanced budget multiplier” in this model? That is, does \(dT=dG=dy\) hold in this model? Demonstrate mathematically and explain why your answer makes sense. (15 points)

As we touched on in class, Keynesian economists and Monetarist economists reach different conclusions regarding the effectiveness of fiscal policy and monetary policy. For example, Keynesians tend to favor expansionary fiscal policy over expansionary monetary policy as a means of “fighting” a recession. On the other hand, Monetarists tend to favor expansionary monetary policy over expansionary fiscal policy as a means of “fighting” a recession. Although you have neither the time nor the mathematical skills to explore the full range of the debate between Keynesian economists and Monetarist economists, you do have the time and the mathematical skills to explore one component of the debate.

b. Consider the money supply multiplier we developed in class.

(1) What happens to the size of this multiplier as investment \((I)\) becomes less responsive to changes in the interest rate \((r)\)? (HINT: Consider what happens to the size of the multiplier as \(I'(r)\) goes to zero.) (10 points)

(2) Suppose investment is independent of the interest rate. What impact will an increase in the money supply have on the level of output in this case? Demonstrate mathematically. What impact will an increase in the money supply have on the interest rate in this case? Demonstrate mathematically. Illustrate your analysis with an ISLM diagram. (NOTE: You need to pay special attention to the slope of the IS curve.) (20 points)

c. Consider the government expenditures multiplier we developed in class.

(1) What happens to the size of this multiplier as investment \((I)\) becomes less responsive to changes in the interest rate \((r)\)? (10 points)

(2) Suppose investment is independent of the interest rate. What impact will an increase in government expenditures have on the level of output in this case? Demonstrate mathematically. What impact will an increase in government expenditures have on the interest rate in this case? Demonstrate mathematically. Illustrate your analysis with an ISLM diagram. (20 points)
4. The ISLM model we considered in class did not include exports or imports. In effect, we considered a “closed economy” ISLM model. The objective of the questions in this section is to consider some of the implications of adding exports and imports to the basic ISLM model.

The aggregate expenditures function in an “open economy” is defined by

\[ e = C(y) + I(r) + G + X - \mu(y_d) \]

where \( X \) represents exports and \( \mu(y_d) \) represents imports. Exports are determined exogenously. Imports depend on the economy’s level of disposable income \( y_d \). The “other” components of the aggregate expenditures function are as defined in class (same notation; same definitions; same properties). Assume \( 0<\mu'(y_d)<1 \). Also, assume \( \mu'(y_d)<C'(y_d) \). Finally, assume \( 0<[\mu'(y_d)+C'(y_d)]<1 \). Equilibrium in the domestic goods market is characterized by

\[ y = C(y) + I(r) + G + X - \mu(y_d). \]

Following the model we developed in class, equilibrium in the money market is characterized by

\[ M_S = M_D(r, y) \]

where \( M_S \) represents the supply of money and \( M_D \) represents the demand for money.

a. What impact will an increase in exports have on the interest rate and the level of output in this open economy? Demonstrate mathematically. (20 points)

b. Derive the government expenditures multiplier for this open economy. Compare this government expenditures multiplier with the government expenditures multiplier we derived in class for a closed economy. Explain how and why these government expenditures multipliers differ. (25 points)

c. Derive the money supply multiplier for this open economy. Compare this money supply multiplier with the money supply multiplier we derived in class for a closed economy. Explain how and why these money supply multipliers differ. (25 points)