PROBLEM SET 1

This problem set is worth 300 points. The point-value of each question is stated in parentheses after the question.

1. Consider the following market demand function

\[ X(P) = aP^\alpha \quad a>0, \alpha>0, P>0 \]

where \( X \) represents the quantity of \( X \) demanded and \( P \) represents the price of \( X \).

a. The domain of this function is restricted to strictly positive prices (\( P>0 \)). Briefly explain why it makes sense to restrict the domain of this demand function in this fashion. What “problems” occur if \( P \leq 0 \)? (NOTE: Be careful here.) (10 points)

b. Present a “rough sketch” of this demand function. Use derivatives to justify your rough sketch. (NOTE: The “usual” demand curve diagram plots \( P \) on the vertical axis and plots \( X \) on the horizontal axis. So, you might want to work with the inverse demand function here.) (20 points)

c. Determine how the own-price elasticity of the demand for \( X \) will change as the price of \( X \) rises. (15 points)

d. Derive an equation that states the consumers’ total expenditures on \( X \) as a function of the price of \( X \). Use this equation to determine how consumers’ total spending on \( X \) will change as the price of \( X \) rises. (NOTE: There will be three cases to consider here.) (15 points)

2. Consider the following market supply function

\[ X(P) = -m + nP^\alpha \quad m>0, n>0, \alpha>0 \]

where \( X \) represents the quantity of \( X \) supplied and \( P \) represents the price of \( X \).

a. What restriction(s) need to be imposed on the domain of this supply function to ensure non-negative supply quantities (\( X \geq 0 \))? (10 points)

b. Present a “rough sketch” of this supply function. Use derivatives to justify your rough sketch. (NOTE: There are three cases to consider here. Present a separate diagram for each case.) (20 points)
3. The market for good X is described by the following demand and supply functions

\[(1) \quad X_S(P_X) = -m + nP_X \quad m>0, n>0\]

\[(2) \quad X_D(P_X, P_Y) = a - bP_X - cP_Y \quad a>0, b>0, c>0, P_Y>0\]

where \(X_S\) represents the quantity of good X supplied, \(P_X\) represents the price of good X, \(X_D\) represents the quantity of good X demanded, and \(P_Y\) represents the price of good Y.

a. How are good X and good Y related in consumption? Briefly explain how you determined this. (10 points)

b. Derive the equilibrium values for the market price of X, denoted by \(P_X^*\), and the market quantity of X, denoted by \(X^*\), as functions of a, b, c, \(P_Y\), m, and n. What restriction(s) do you need to impose to ensure \(P_X^*>0\) and \(X^*>0\)? Present a demand and supply diagram to illustrate. (HINT: You might want to “work backwards” here. That is, draw you diagram first and see what restriction(s) you need to ensure \(P_X^*>0\) and \(X^*>0\). Then, relate the necessary restriction(s) to the equations you derived for \(P_X^*\) and \(X^*\). (20 points)

c. Use the solutions for \(P_X^*\) and \(X^*\) from part b to derive and sign the following comparative statics results. (20 points)

\[\frac{\partial X^*}{\partial P_Y}; \frac{\partial P_X^*}{\partial P_Y}; \frac{\partial X^*}{\partial n}; \frac{\partial P_X^*}{\partial n}\]

d. Present a completely labeled demand and supply diagram to illustrate how the equilibrium price of X and the equilibrium quantity of X will change in response to a decrease in \(P_Y\). Briefly explain how this diagram relates to the comparative statics results you derived in part c. (15 points)

e. Present a completely labeled demand and supply diagram to illustrate how the equilibrium price of X and the equilibrium quantity of X will change in response to an increase in n. Briefly explain how this diagram relates to the comparative statics results you derived in part c. (15 points)
4. The market for jeans in Dipsville is described by the following demand and supply functions

(1) \( J_D = J_D(P_J, N(A)) \)

(2) \( J_S = J_S(P_J, w_D) \)

where \( J_D \) represents the quantity of jeans demanded, \( P_J \) represents the price of jeans, \( N \) represents the number of demanders of jeans in Dipsville, \( A \) represents the average age of a citizen of Dipsville, \( J_S \) represents the quantity of jeans supplied, and \( w_D \) represents the price of (unfinished) denim. Note that \( N \) is a function of \( A \). Assume \( N'(A) < 0 \). (NOTE: In this simple model, we are assuming the size of the population is fixed.) In addition, assume \( \partial J_D/\partial P_J < 0 \) and \( \partial J_S/\partial P_J > 0 \).

a. Identify the endogenous variables in this model. Identify the exogenous variables in this model. Explain why it is important to distinguish between endogenous variables and exogenous variables. (10 points)

b. Assign signs to the following partial derivatives. Briefly justify each of these signs.

\[
\frac{\partial J_D}{\partial A}, \frac{\partial J_S}{\partial w_D}
\]

c. Derive and sign the following comparative statics results. (20 points)

\[
\frac{dP_J}{dA}; \frac{dJ}{dA}; \frac{dP_J}{dw_D}; \frac{dJ}{dw_D}
\]

d. Present a “rough sketch” demand and supply diagram to illustrate how the market price and market quantity of jeans will change as the citizens of Dipsville get older on average, ceteris paribus. Explain how this diagram relates to the comparative statics results you derived in part c. (10 points)

e. Present a “rough sketch” demand and supply diagram to illustrate how the market price and market quantity of jeans will change in response to an increase in the price of (unfinished) denim, ceteris paribus. Explain how this diagram relates to the comparative statics results you derived in part c. (10 points)
5. The initial market demand for good X is given by

\[ X_D(P_X, P_Z) = 4500 - 50P_X + 20P_Z \]

where \( X_D \) represents the quantity of good X demanded, \( P_X \) represents the price of good X, and \( P_Z \) represents the price of good Z. The initial market supply of good X is given by

\[ X_S(P_X) = 20P_X \]

where \( X_S \) represents the quantity of good X supplied.

The initial market demand for good Z is given by

\[ Z_D(P_Z, P_X) = 1500 - 40P_Z + 25P_X \]

where \( Z_D \) represents the quantity of good Z demanded. The initial market supply of good Z is given by

\[ Z_S(P_Z) = 10P_Z \]

where \( Z_S \) represents the quantity of good Z supplied.

a. Derive the equilibrium values for \( P_X, X, P_Z, \) and \( Z \). (10 points)

Good X is produced domestically and good Z is imported from other countries. Suppose the government imposes a per-unit import tax on the firms that import good Z and at the same time awards a per-unit consumption subsidy to the buyers of good X. Let \( t_Z \) represent the per-unit import tax on good Z. Let \( s_X \) represent the per-unit consumption subsidy on good X. (NOTE: The import tax on good Z is collected directly from the firms that import good Z and the consumption subsidy on good X is awarded directly to the buyers of good X.)

b. Derive the equilibrium values for \( P_X, X, P_Z, \) and \( Z \) as functions of \( t_Z \) and \( s_X \). (NOTE: You need to rewrite the demand and supply functions to account for \( t_Z \) and \( s_X \).) (20 points)

THIS SECTION CONTINUES ON THE NEXT PAGE.
c. Derive and sign the following comparative statics results. (20 points)

\[
\frac{dP_X}{dt_Z}; \frac{dP_X}{ds_X}; \frac{dX}{dt_Z}; \frac{dX}{ds_X}
\]

\[
\frac{dP_Z}{dt_Z}; \frac{dP_Z}{ds_X}; \frac{dZ}{dt_Z}; \frac{dZ}{ds_X}
\]

d. According to the comparative statics results you derived in part c, how will an increase in the import tax on good Z \((t_Z)\) affect \(P_X, X, P_Z,\) and \(Z\)? Present “rough sketch” demand and supply diagrams to explain why these results make sense. (10 points)

e. According to the comparative statics results you derived in part c, how will an increase in the consumption subsidy on good X \((s_X)\) affect \(P_X, X, P_Z,\) and \(Z\)? Present “rough sketch” demand and supply diagrams to explain why these results make sense. (10 points)