PROBLEM SET 5

This problem set is worth 250 points. The point-value of each question is stated in parentheses after the question.

1. Consider a profit maximizing, perfectly competitive firm that produces output (X) subject to the production function

   \[ X(v_1, v_2) = 10v_1^{1/2}v_2^{1/2} \]

where \( v_1 \) represents units of input 1 and \( v_2 \) represents units of input 2. Assume \( v_2 \) is fixed at 36 units in the short run (\( \nabla_2 = 36 \)). Therefore, the firm’s short-run production function is given by

   \[ X(v_1) = 60v_1^{1/2} \].

   The short-run marginal product of \( v_1 \) is given by

   \[ \text{MP}_1(v_1) = 30/v_1^{1/2}. \]

   a. Derive the firm’s short-run input demand for \( v_1 \). This function will take the general form \( v_1 = v_1(w_1, P) \), where \( w_1 \) is the per-unit price of \( v_1 \) and \( P \) is the per-unit price of \( X \). Derive the firm’s short-run output supply of \( X \). This function will take the general form \( X = X(w_1, P) \). You may employ these functions when answering the following questions in this section. (10 points)

   Assume the initial per-unit market price of \( X \) is $20 (\( P_0 = 20 \)), the initial per-unit price of \( v_1 \) is 20 (\( w_1^0 = 20 \)), and the initial per-unit price of \( v_2 \) is 100 (\( w_2^0 = 100 \)).

b. Given these initial prices, how many units of \( v_1 \) will this firm employ in the short run, how many units of output will this firm produce in the short run, and how much profit will this firm earn in the short run? (10 points)

c. Use marginal analysis to explain IN DETAIL why this firm will employ neither more nor less than the number of units of \( v_1 \) you identified in part b. Illustrate your analysis with a marginal revenue product (MRP) diagram. (REMINDER: The MRP curve also represents the firm’s input demand for \( v_1 \)). (10 points)

d. Suppose the market price of \( v_1 \) rises from \( w_1^0 = 20 \) to \( w_1^1 = 30 \), ceteris paribus. How will this firm’s employment of \( v_1 \), output of \( X \), and profit change as a result of this increase in \( w_1 \)? Explain why the firm’s \( v_1 \) employment response “makes sense” in this case. Illustrate with a \( v_1 \) demand diagram. (10 points)
e. In my intro micro course, I usually discuss how a profit-maximizing firm will alter its output in response to an increase in the wage of labor (a variable input in the short run). This discussion confuses many of my intro micro students. In fact, I often see variants of the following statement on my intro micro exams:

If the wage of labor rises, then the firm will produce more output in order to generate the additional revenue it needs to cover the increased labor costs.

Given your analysis in part d, explain why it will not be in the best interest of a profit-maximizing firm to increase its output when the wage of labor rises. (10 points)

f. Return to the initial set of prices ($P_0=20$, $w_1^0=20$, $w_2^0=100$). Suppose the market price of the firm’s output falls from $P_0=20$ to $P_1=10$, ceteris paribus. How will this firm’s employment of $v_1$, output of $X$, and profit change as a result of this decrease in $P$? Explain why the firm’s $v_1$ employment response “makes sense” in this case. Illustrate with a $v_1$ demand diagram. (10 points)

g. Many of my intro micro students are confused here, too. I often see variants of the following statement on my intro micro exams:

If the price of output falls, then a profit-maximizing firm will sell more output in order to make up for the revenue it loses as a result of the lower output price.

Given your analysis in part f, explain why it will not be in the best interest of a profit-maximizing firm to increase its output when the price of its output falls. (10 points)

Return to the original equilibrium you derived in part b. Now, suppose the firm finds itself with only 25 units of $v_2$ in the short run. If $\bar{v}_2=25$, then the short-run production function is given by

\[ X(v_1) = 50v_1^{1/2} \]

and the short-run marginal product of $v_1$ is given by

\[ MP_1(v_1) = 25/v_1^{1/2}. \]

h. How will this firm’s employment of $v_1$, supply of $X$, and profit change as a result of this decrease in the firm’s employment of $v_2$? Illustrate the $v_1$ employment effect with a $v_1$ demand diagram. Illustrate the output supply effect with an output supply diagram. Explain why the $v_1$ input demand response and the output supply response “make sense” in this case. (10 points)
2. Consider a firm that employs male workers (M) and female workers (F) to produce its output (X). The firm’s production technology is characterized by downward sloping, strictly convex isoquants. Assume males and females are gross substitutes in production.

The firm is a perfect competitor in the output market and in both labor markets. Let $P$ represent the market price of the firm’s output. Let $w_M$ represent the market wage of a male worker and let $w_F$ represent the market wage of a female worker.

The owner of the firm incurs a monetary cost of $w_F$ when it employs a female worker. However, the owner of the firm “dislikes” employing females and therefore behaves as if he also incurs a “psychic” cost when he employs a female worker. Therefore, the owner of the firm behaves as if the marginal factor cost of female labor is

$$ MFC_F = w_F + \alpha_F $$

where $\alpha_F > 0$ is the monetary equivalent of the psychic cost the owner of the firm incurs when he employs a female worker. The owner of the firm does not incur a psychic cost when he employs a male worker. Therefore, the owner of the firm behaves as if the marginal factor cost of male labor is

$$ MFC_M = w_M. $$

The total cost of employing labor (monetary plus psychic) can be expressed as

$$ CT = w_MM + (w_F + \alpha_F)F. $$

a. Does this firm have the ability to “exploit” the females it employs by paying them a wage that is less than the market wage of female labor? Explain briefly. (5 points)

b. The discriminating firm will employ females and males so as to minimize the total cost of producing a given level of output ($X_0$). Employ an isocost/isoquant diagram to demonstrate that this firm does not minimize the monetary cost of producing a given level of output ($X_0$). Put $F$ on the horizontal axis of your diagram. (10 points)

c. How would this firm’s labor demands compare to the labor demands in a firm that does not discriminate against females? Explain. Illustrate with labor demand curves. (HINTS: Start with the labor demands that would exist in a non-discriminating firm ($\alpha_F=0$), and then consider the labor demand effects of discrimination against females.) (NOTE: You can, and probably should, address the issues raised here without employing any isocost/isoquant diagrams. However, you will need to consider substitution, output, and total effects in your analysis.) (15 points)
Parts d, e, and f explore some of the market-level effects of employer discrimination against women. Keep in mind that your analysis in parts a, b, and c applies to the behavior of an individual firm, and as such does not apply directly to a market-level analysis of employer discrimination against women.

d. How will the market wage and the market employment of females and males be affected in the short run if a substantial number of firms in the labor market discriminate against females as modeled in parts a, b, and c above? Illustrate and explain. (HINT: Begin your analysis from a hypothetical non-discriminatory equilibrium.) (10 points)

e. Would you expect the labor market effects of employer discrimination against females to dissipate over time? Why or why not? (NOTE: You MUST answer this question within the context of the employer discrimination model being developed in this section.) (SUGGESTION: Assume perfectly competitive markets here.) (10 points)

f. What does your analysis in parts d and e above imply about the necessity of enforcing anti-discrimination laws in the long run? Discuss. (NOTE: The is NOT a “personal opinion” question. I am asking you to consider a policy implication of this model.) (10 points)
3. Two firms (F₁ and F₂) sell differentiated products. The market demand for Firm 1’s output is given by

\[ X_1(P_1, P_2) = 400 - 20P_1 + 10P_2 \]

where \( X_1 \) represents Firm 1’s output, \( P_1 \) represents the price Firm 1 charges for its output, and \( P_2 \) represents the price Firm 2 charges for its output. The cost of production in Firm 1 is given by

\[ C_1(X_1) = 10X_1. \]

Firm 1 chooses its price from the following action set

\[ A_1 = \{10, 15, 20, 25\}. \]

The market demand for Firm 2’s output is given by

\[ X_2(P_1, P_2) = 400 + 10P_1 - 20P_2 \]

where \( X_2 \) represents Firm 2’s output. The cost of production in Firm 2 is given by

\[ C_2(X_2) = 10X_2. \]

Firm 2 chooses its price from the following action set

\[ A_2 = \{10, 15, 20, 25\}. \]

a. Construct the payoff matrix for this game. (10 points)

Suppose the firms know the payoff matrix you constructed in part a. Further, suppose the firms choose their respective prices simultaneously and independently.

b. What outcome would you predict in this game? Justify your proposed solution. Is your proposed solution a Nash equilibrium in prices? Explain. (15 points)

In class we briefly considered a pricing game in which the two competing firms were selling homogeneous goods and faced identical marginal and average costs of production. In that game, we predicted that price competition between the two firms would drive the price to marginal cost.

c. Does this conclusion hold for the game under consideration here? Discuss the economic intuition underlying your conclusion. (10 points)
Now, suppose we change the nature of the game a little bit. Firm 1 sets and announces its price first. Firm 1 is the “price leader.” Then Firm 2 sets and announces its price. Firm 2 is the “price follower.” Firm 2 knows the price set by Firm 1 when it comes time for Firm 2 to set its price. In addition, both firms know the payoff matrix.

d. Discuss Firm 1’s decision-making process in this game. (10 points)

e. What outcome would you predict in this game? Justify your proposed solution. (NOTE: A game tree might be very helpful here.) (15 points)

f. Which firm would you rather be in this “leader-follower” game, Firm 1 or Firm 2? Discuss your reasoning. (10 points)
Three legislators (\(L_1\), \(L_2\), \(L_3\)) must vote on whether to give themselves a pay raise. All three want the pay raise. Let \(B\) represent the benefit each legislator derives from getting the raise. However, each legislator also faces a cost in terms of voter resentment if he/she votes for the pay raise. Let \(C\) represent this cost. Assume \(B > C\). The pay raise will pass only if two or more legislators vote for the pay raise. Let \(F\) represent a vote in favor of the pay raise. Let \(A\) represent a vote against the pay raise.

The legislators vote sequentially (\(L_1\) votes first, \(L_2\) votes second, and \(L_3\) votes last). A legislator’s vote is “public knowledge” the instant it is cast.

a. Construct the game tree for this voting game. Be sure to list the payoffs \((\pi_1, \pi_2, \pi_3)\) at each of the game’s terminal nodes. (10 points)

b. Is it better to vote first or vote last? Discuss your reasoning. (20 points)