PROBLEM SET 3

This problem set is worth 200 points. The point-value of each question is stated in parentheses after the question.

Consider an exchange-only economy consisting of two traders (A and B) and two goods (X₁ and X₂). The preferences of A and B over X₁ and X₂ are represented by the following utility functions:

(1) \( U^{A}(X_{1}^{A},X_{2}^{A}) = 10(\min\{5X_{1}^{A},2X_{2}^{A}\}) + 20 \)
(2) \( U^{B}(X_{1}^{B},X_{2}^{B}) = 15(\min\{X_{1}^{B},X_{2}^{B}\}) + 12 \)

where \( X_{1}^{A} \) and \( X_{2}^{A} \) represent A's consumption of \( X_{1} \) and \( X_{2} \), respectively, and \( X_{1}^{B} \) and \( X_{2}^{B} \) represent B's consumption of \( X_{1} \) and \( X_{2} \), respectively.

Trader A's initial endowment of \( X_{1} \) and \( X_{2} \) is given by

(3) \( \bar{X}^{A} = (\bar{X}_{1}^{A}, \bar{X}_{2}^{A}) = (16,4) \).

Trader B's initial endowment of \( X_{1} \) and \( X_{2} \) is given by

(4) \( \bar{X}^{B} = (\bar{X}_{1}^{B}, \bar{X}_{2}^{B}) = (2,20) \).

a. Consider the preferences for trader A and ignore A's endowment for the moment. Draw a set of representative indifference curves (2 ought to be sufficient) that are consistent with A’s preferences as represented by the utility function in equation 1. Put \( X_{1} \) on the horizontal axis. Demonstrate these indifference curves do in fact represent A’s preferences. Connect the indifference curve “kinks” with a dotted line. This dotted line is called an “expansion path.” State the equation for A’s expansion path. (NOTE: This equation will take the general form \( X_{2}^{A} = f(X_{1}^{A}) \).) (10 points)

b. On a “new” diagram for trader A, locate (i) A’s initial endowment, (ii) the indifference curve that goes through A’s initial endowment (\( \bar{I}^{A} \)), and (iii) A’s expansion path. Graphically derive trader A's offer curve (\( \bar{e}^{A} \)). Be sure to provide an explanation to go along with your graphical derivation. (HINT: Begin from a situation in which the budget line through A's initial endowment is horizontal (\( \rho_{1}=0 \)). In this case, A is willing to make any trade that lies along the “lower leg” of \( \bar{I}^{A} \). Now, let the relative price of \( X_{1} \) rise “a little bit.” What trade is A willing to make now? Let the relative price of \( X_{1} \) rise “a little bit more.” What trade is A willing to make now? Keep raising the relative price of \( X_{1} \) until the budget line through A's initial endowment is vertical (\( \rho_{1}=\infty \)). What trade(s) is A willing to make now? After you have done all this, you should be able to identify A’s offer curve!!!) (20 points)
c. Repeat part a for trader B. (NOTE: The equation for trader B's expansion path will take the general form \( X_2^b = g(X_1^b) \).) (10 points)

d. Repeat part b for trader B. Label the indifference curve that goes through B's initial endowment as \( I^b \). Label B's offer curve as \( e^b \). (HINT: Start your analysis with \( \rho_1 = \infty \), and then keep dropping the relative price of \( X_1 \) until \( \rho_1 = 0 \).) (NOTE: No explanation is needed here.) (10 points)

You will need to draw Edgeworth box diagrams for parts e, f, and g. Put \( X_1 \) on the horizontal edges of the box and put \( X_2 \) on the vertical edges of the box. Put trader A's origin at the lower left-hand corner of the box. Put trader B's origin at the upper right-hand corner of the box. Make sure your diagrams are BIG, NEAT, and COMpletely LABELED.

e. Locate the set of pareto optimal allocations for this economy. Justify your conclusion. (REMINDER: The pareto set is not defined in relation to the initial endowment allocation.) (NOTE: You should present more than one diagram here.) (WARNING: Avoid "proof by proclamation.") (30 points)

f. Draw a "new" Edgeworth box for this economy. Add the following to your diagram:
- the initial endowment allocation; label this as \( X \)
- \( I^A \) and \( I^B \)
- A's offer curve (\( e^A \)) and B's offer curve (\( e^B \)).

(i) Where is the equilibrium allocation located? I want "specific numbers" here. (REMINDER: An allocation in this economy is stated as \( X = (X_1^A, X_2^A, X_1^B, X_2^B) \).) Explain the economic intuition underlying your solution technique. Add the equilibrium allocation to your diagram. Label this allocation as \( X^* \). (20 points)

(ii) What is the equilibrium relative price of \( X_1 \) equal to? Again, I want a "specific number" here. Add this relative price to your diagram. Label this relative price as \( \tilde{n}_1^* \). (NOTE: No explanation/discussion is needed here.) (10 points)

(iii) Is the equilibrium allocation you identified in (i) pareto optimal? How do you know this? (10 points)

(iv) Do both traders gain from exchange in this economy? How do you know this? (10 points)

g. Consider the equilibrium relative price of \( X_1 \) you identified in part f (ii). Explain, IN DETAIL, why no other relative price of \( X_1 \) can be an equilibrium relative price in this economy. (NOTE: You will need at least one "new" diagram here.) (Warning: You MUST apply the formal definition of general equilibrium here.) (40 points)
A feasible allocation \((X_1^A, X_2^A, X_1^B, X_2^B)\) is said to be **equitable** if

\[ U^A(X_1^A, X_2^A) \geq U^A(X_1^B, X_2^B) \quad \text{and} \quad U^B(X_1^B, X_2^B) \geq U^B(X_1^A, X_2^A). \]

h. Interpret equation 5. (10 points)

A feasible allocation is said to be **fair** if it is both equitable and pareto optimal.

i. Is the equilibrium allocation you identified above in part f (i) fair? Justify your answer. (20 points)