PROBLEM SET 2

This problem set is worth 350 points. The point-value of each question is stated in parentheses after the question.

1. Consider an individual who lives two (time) periods, the present \( t=1 \) and the future \( t=2 \). Let \( M_1 \) represent his income in the current period. Let \( M_2 \) represent his income in the future period. The consumer spends his income on a composite consumption commodity. Let \( C_1 \) represent his consumption spending in the current period. Let \( C_2 \) represent his consumption spending in the future period.

The consumer’s two-period spending constraint is given by

\[
C_1 + C_2/(1+r) = M_1 + M_2/(1+r)
\]

where \( r \) is the interest rate \( (0<r<1) \). The left-hand-side of equation 1 is the present value of total consumption spending in the two periods. The right-hand-side of equation 1 is the present value of total income in the two periods.

a. Briefly explain why it is necessary to “discount” \( C_2 \) and \( M_2 \) by \( (1+r) \). (10 points)

b. Diagram the consumer’s two-period budget set. Put \( C_1 \) on the horizontal axis and \( C_2 \) on the vertical axis. Be sure to label the intercepts and the slope of the budget line. Also, be sure to locate the point \((M_1,M_2)\) on your diagram. (10 points)

c. On the diagram you drew for part b, locate (and label) a consumption bundle such that \( C_1>M_1 \). (NOTE: Do not add any indifference curves yet.) Briefly explain how this consumption bundle is possible. On the same diagram, locate (and label) a consumption bundle such that \( C_2>M_2 \). (NOTE: Do not add any indifference curves yet.) Briefly explain how this consumption bundle is possible. (10 points)

d. What is the relative price of current consumption \( (C_1) \) equal to? Briefly explain why this relative price “makes sense” within the framework of the model being developed here. (10 points)

The consumer’s preferences over \( C_1 \) and \( C_2 \) are represented by

\[
U(C_1,C_2) = u(C_1) + \delta u(C_2)
\]

where \( \delta \) is the consumer’s discount factor \( (0<\delta<1) \). For simplicity, assume this utility function generates downward sloping, non-intersecting, strictly convex indifference curves.

e. Briefly explain why it is necessary to discount \( u(C_2) \) by \( \delta \). (10 points)
f. On a “new” diagram, illustrate a representative indifference curve. Put $C_1$ on the horizontal axis and $C_2$ on the vertical axis. How would you interpret the slope of the indifference curve. (NOTE: Do not include a budget line here.) (HINT: Think about “time preferences.”) (10 points)

g. Under what condition(s) would the consumer choose a two-period pattern of consumption such that $C_1 > M_1$? Illustrate this case with a “new” budget line/indifference curve diagram. Under what condition(s) would the consumer choose a two-period pattern of consumption such that $C_1 < M_1$? Illustrate this case on the same diagram. (10 points)

We are FINALLY ready for some application questions!

h. Consider the case in which $C_1 > M_1$. How will the consumer change his current consumption in response to a decrease in the interest rate? Illustrate your analysis with a “new” budget line/indifference curve diagram. Be sure to provide an explanation to go along with your diagram. (HINT: Every budget line must go through the point $(M_1, M_2)$. Why?) (15 points)

i. Consider the case in which $C_1 < M_1$. How will the consumer change his current consumption in response to a decrease in the interest rate? Illustrate your analysis with a “new” budget line/indifference curve diagram. Be sure to provide an explanation to go along with your diagram. (15 points)

j. The Fed is in the process of driving US interest rates down in an attempt to stimulate aggregate demand in the US economy. According to many “experts,” the decrease in interest rates will generate a significant increase in current consumption demand and therefore a significant increase in current aggregate demand. Evaluate this prediction in light of your analysis in parts h and i. (NOTE: Consumption spending constitutes approximately 66% of total spending in the US economy.) (REMINDER: Not all consumers are identical. For some consumers, $C_1 > M_1$ and for other consumers, $C_1 < M_1$.) (15 points)
2. Buffy allocates her time \((T)\) between hours of work for pay in the labor market \((L)\) and hours spent in the household \((H)\). Her time constraint is defined by

\[
T = L + H.
\]

Her only source of income is the money she earns in the labor market. She spends her labor market income on a single market good \((X)\). Her spending constraint is defined by

\[
P_X X = wL
\]

where \(P_X\) is the market price of good \(X\) and \(w\) is Buffy’s hourly market wage. For simplicity, assume \(P_X = 1\). Substituting \(L = T - H\) from equation 1 into equation 2 and setting \(P_X = 1\) yields the equation for Buffy’s budget line

\[
X = wT - wH.
\]

Buffy’s preferences over \(X\) and \(H\) are well behaved. Assume \(X\) and \(H\) are income normal.

Suppose the government taxes Buffy’s labor market income. Buffy’s (total) income tax bill \((TB)\) is determined by the following tax function:

\[
TB = \begin{cases} 
0 & \text{if } wL \leq wL' \\
\tau_1(wL - wL') & \text{if } wL > wL'
\end{cases}
\]

where \(0 < \tau_1 < 1\) and \(0 < L' < T\).

a. How does the imposition of this income tax affect Buffy’s budget set? Illustrate with a completely labeled budget set diagram. Put \(H\) on the horizontal axis and \(X\) on the vertical axis. (NOTE: Be sure to include an original, pre-tax budget line. Do not include any indifference curves in this diagram.) (REMINDER: \(L = T - H\).) (10 points)

b. Employ the diagram you drew for part a to discuss the properties of the income tax system defined by the tax function in equation 4. (10 points)

c. Suppose Buffy supplies \(L_1\) hours to the labor market under this income tax system. Illustrate, on a “new” diagram, the solution to Buffy’s choice problem when \(L' < L_1 < T\). Locate and label the optimal values for hours spent in the household \((H_1)\), hours spent in the labor market \((L_1)\), before-tax labor market income \((wL_1)\), after-tax labor market income \((wL_1 - \tau_1(wL_1 - wL'))\), and the total income tax bill \((\tau_1(wL_1 - wL'))\). (10 points)
The government is thinking about replacing the current income tax system defined by the tax function in equation 4 with a new income tax system under which all labor market income will be taxed at the same rate \( \tau \), where \( 0 < \tau < 1 \). The government wants to set \( \tau \) such that the total tax revenue generated by the new income tax system will be equal to the total tax revenue generated by the current income tax system. In other words, the government wants the new income tax system to be “revenue neutral.”

Bubba is on your research staff at Hekifweno Research Associates. You have asked him to determine the revenue neutral value for \( \tau \). Bubba has submitted the following policy recommendation to you for your evaluation.

**Bubba’s Policy Recommendation**

Consider a representative worker whose current market wage is equal to the average wage in the labor market (\( \bar{w} \)) and whose current labor market hours are equal to the average hours of work in the labor market (\( \bar{L} \)). Further, suppose the representative worker’s marginal tax rate under the current income tax system is \( \tau_1 \). Under the current income tax system, the representative worker’s income tax bill is equal to \( \tau_1 (\bar{w}L - \bar{w}L') \), where \( L' < \bar{L} < T \). Under the new income tax system, the representative worker’s income tax bill will be equal to \( \tau \bar{w}L \). The new income tax system will be revenue neutral for this representative worker when \( \tau \bar{w}L = \tau_1 (\bar{w}L - \bar{w}L') \). This implies \( \tau = \tau_1 [1 - (L'/\bar{L})] \) is the revenue neutral tax rate for the representative worker.

If the government sets \( \tau = \tau_1 [1 - (L'/\bar{L})], \) then

1. workers whose current income tax bill is greater than the current income tax bill of the representative worker will see their income tax payments fall under the new income tax system, and

2. workers whose current income tax bill is less than the current income tax bill of the representative worker will see their income tax payments rise under the new income tax system.

Since the representative worker, as defined above, is the “average worker,” these two opposing tax revenue effects will be similar in magnitude if the government sets \( \tau = \tau_1 [1 - (L'/\bar{L})] \) under the new income tax system.

Therefore, in order to achieve revenue neutrality, the government should set \( \tau = \tau_1 [1 - (L'/\bar{L})] \) under the new income tax system.

d. Evaluate Bubba’s policy recommendation. Employ diagrams where appropriate. (30 points)
3. Zippy consumes only two goods, $X_1$ and $X_2$. His preferences over these two goods are summarized by the utility function

\[(1) \quad U(X_1, X_2) = \alpha_1 X_1 + \alpha_2 X_2\]

where $\alpha_1 > 0$ and $\alpha_2 > 0$.

It takes money and ration coupons to purchase these goods. Zippy must pay $P_1$ dollars and $\rho_1$ ration coupons to purchase a unit of $X_1$. Similarly, Zippy must pay $P_2$ dollars and $\rho_2$ ration coupons to purchase a unit of $X_2$. Zippy’s money constraint is given by

\[(2) \quad P_1 X_1 + P_2 X_2 = M\]

where $M$ represents Zippy’s money income. Zippy’s ration coupon constraint is given by

\[(3) \quad \rho_1 X_1 + \rho_2 X_2 = R\]

where $R$ is Zippy’s endowment of ration coupons. Assume $(R/\rho_2) > (M/P_2)$ and $(M/P_1) > (R/\rho_1)$. (NOTE: Zippy does not pay for his ration coupons. A ration coupon can be used to purchase either $X_1$ or $X_2$.)

a. Diagram Zippy’s budget set. Put $X_1$ on the horizontal axis and $X_2$ on the vertical axis. Completely label your diagram. (NOTE: Your budget line should have a “kink” in it. You need to derive the values of $X_1$ and $X_2$ at the kink.) Explain why the consumption bundles that lie outside Zippy’s budget set are not available to Zippy. (15 points)

b. What is the relative price of $X_1$ equal to? Explain. (10 points)

c. State the condition under which Zippy’s optimal consumption bundle will be the bundle at which the budget line kinks. (HINT: This condition can be stated in terms of $\alpha_1$, $\alpha_2$, $\rho_1$, $\rho_2$, $P_1$, and $P_2$ and will be in the form of a three-term inequality.) Illustrate this condition with a budget line/indifference curve diagram. (10 points)

d. Assume Zippy’s optimal consumption bundle lies at the budget line kink. Now, suppose the government reduces the number of ration coupons allocated to Zippy from $R$ to $R'$, where $(R'/\rho_2) > (M/P_2)$. How and why will this affect Zippy’s consumption of $X_1$ and $X_2$? Illustrate with a “new” budget line/indifference curve diagram. (10 points)
4. Billy Beerbelly allocates his money income (M) between beer (B) and spending on all other goods (AOG). His preferences over B and AOG are summarized by the utility function

\( U(B, AOG) = 18 \ln B + 2AOG \)

where \( \ln B \) is the natural logarithm of the number of beers consumed by Billy. Billy’s budget constraint is given by

\( M = P_B B + AOG \)

where \( P_B \) is the price of a beer. Billy’s demand for beer is given by

\( B_D(P_B) = (9/P_B). \)

a. Briefly explain why this demand function is not particularly realistic. Illustrate with a budget line/indifference curve diagram. Put B on the horizontal axis and AOG on the vertical axis. (10 points)

b. How many beers will Billy consume and how much money will he spend on all other goods when his money income is $90 (M=90) and the price of a beer is $3 (\( P_B = 3 \))? (5 points)

c. There are two bars in town: Beverly’s Beer Garden (BBG) and The Hole in the Wall Tavern (HWT). They are in the middle of a “price war.” The price of beer at BBG is $3. HWT has just reduced its price of beer from $3 to $1.50. Billy views a beer at BBG as being a perfect substitute for a beer at HWT. Where will Billy drink beer? How do you know this? How many beers will Billy drink and how much money will he spend on all other goods? (10 points)

d. BBG keeps its price at $3 a beer. HWT keeps its price at $1.50 a beer. However, HWT introduces a “cover charge” that Billy must pay before he is allowed to purchase beer at HWT. What is the maximum cover charge Billy is willing to pay to get into HWT? Show all your work. Present a budget line/indifference curve diagram to support your conclusion. (15 points)
5. Bobbi Jo allocates her income (M) between gropnooks (G) and blubnuts (B). Let $P_G$ represent the market price of gropnooks. Let $P_B$ represent the market price of blubnuts. For now, assume $P_G>0$ and $P_B>0$.

Bobbi Jo’s preferences over gropnooks and blubnuts are complete, reflexive, transitive and strictly convex. However, her preferences over gropnooks and blubnuts are not strictly monotone. Although an increase in the consumption of gropnooks, ceteris paribus, makes Bobbi Jo better off, an increase in the consumption of blubnuts, ceteris paribus, makes Bobbi Jo worse off.

a. Diagram a representative indifference curve that is consistent with Bobbi Jo’s preferences over gropnooks and blubnuts. Put blubnuts on the horizontal axis and gropnooks on the vertical axis. Demonstrate that this indifference curve is in fact consistent with Bobbi Jo’s preferences. (NOTE: You must justify the slope and the shape of this indifference curve.) (15 points)

b. What bundle will Bobbi Jo choose as her best available bundle? Justify your conclusion. Illustrate with a budget line/indifference curve diagram. (10 points)

c. On a “new” diagram, demonstrate that Bobbi Jo could be induced to consume some blubnuts if the market price of blubnuts is negative ($P_B<0$). Discuss the economic intuition underlying this result. (NOTE: Be sure to explain what a “negative price” means in this setting.) Will she necessarily consume blubnuts when $P_B<0$? Discuss the economic intuition underlying this result. (15 points)

d. Is Bobbi Jo’s budget set well defined when $P_B<0$? Explain. What “problem(s)” would this create if Bobbi Jo’s preference over B and G were well behaved? Illustrate with a “new” budget line/indifference curve diagram. (10 points)
6. Pookie allocates his income (M) between health care (H) and spending on all other goods (AOG). For simplicity, assume health care is measured in terms of “standardized units.” Let \( P_H \) represent the per-unit price of health care. Pookie’s spending constraint is given by

\[
M = P_H H + AOG.
\]

Pookie’s preferences over H and AOG are well behaved. Assume H and AOG are income normal.

Pookie has the opportunity to purchase a health insurance policy at a price of I dollars. If he purchases the insurance policy, then he can purchase health care at a per-unit price of \( \theta P_H \), where \( 0 < \theta < 1 \). His insurance company will pay \( (1-\theta)P_H \) for each unit of health care Pookie purchases. (NOTE: This payment arrangement is sometimes referred to as a “co-payment” schedule and is quite common in the health insurance industry. For example, an individual who has health insurance might pay a doctor only 20% of the price the doctor charges for an office visit, and the patient’s insurance company will pay the doctor the remaining 80% of the price the doctor charges for an office visit.)

a. Graph Pookie’s budget set if he does not purchase health insurance. Put H on the horizontal axis and AOG on the vertical axis. On the same diagram, graph Pookie’s budget set if he purchases health insurance. Assume \( I < (1-\theta)M \). Briefly explain why you need to assume \( I < (1-\theta)M \). (15 points)

b. Under what condition(s) will Pookie choose to purchase health insurance? Illustrate and explain. How will Pookie alter his consumption of health care if he chooses to purchase health insurance? Illustrate and explain (10 points)

Assume Pookie does in fact purchase health insurance for I dollars and consumes \( H_0 \) units of health care. Now, suppose the government announces that it will provide \( H_G \) units of health care to Pookie at no charge to him. However, if Pookie enrolls in the government health care program, then he can no longer purchase health care in the private market.

c. On a “new” diagram, show it is possible that the availability of “free” government-provided health care might lead to a decrease in Pookie’s consumption of health care. Discuss the economic intuition underlying this result. (NOTE: There might be more than one case to consider here.) (15 points)