PROBLEM SET 1

This problem set is worth 200 points. The point-value of each question is stated in parentheses after the question.

1. You own the cable television company in Ziptodo, Iowa. It is your job to set the monthly cable television fee. You may set your price as you wish. However, you must charge the same price to each of your customers. The monthly demand for cable television service in Ziptodo is given by

\[ C_D = 10000 - 50P_C + 20P_M \]

where \( C_D \) is the number of cable television subscriptions demanded in Ziptodo, \( P_C \) is the monthly fee for a cable television subscription in Ziptodo, and \( P_M \) is the rental price of a movie rental at Marvin’s Movie Mecca, which is the only video store in Ziptodo. You incur a (fixed) cost of $500,000 per month, regardless of how many people in Ziptodo actually subscribe to your cable television service.

a. Define the own-price elasticity of the demand for cable television in Ziptodo. What does this elasticity measure? Explain. Calculate the own-price elasticity of the demand for cable television in Ziptodo when \( P_C^0 = 50 \) and \( P_M^0 = 10 \). Interpret your result. (15 points)

b. Define the cross-price elasticity of the demand for cable television in Ziptodo with respect to the rental price of a movie video at Marvin’s Movie Mecca. What does this elasticity measure? Explain. Calculate the cross-price elasticity of the demand for cable television in Ziptodo with respect to the rental price of a movie video at Marvin’s Movie Mecca when \( P_C^0 = 50 \) and \( P_M^0 = 10 \). Interpret your result. (15 points)

c. What price should you charge for a month of cable television service when \( P_M^0 = 10 \) if your goal is to maximize your profit? Explain why this price “makes sense” in light of the relevant own-price elasticities of the demand for cable television in Ziptodo. (NOTE: You MUST employ elasticity-related concepts/arguments here. Simply calculating and comparing profits at different prices is NOT sufficient, unless you calculate profits at ALL POSSIBLE PRICES.) (15 points)

d. Marvin has announced that he is considering dropping the rental price of a movie video to $5 (\( P_M^1 = 5 \)). How much, if anything, would you be willing to pay Marvin each month to convince him to keep charging $10 for a movie video rental? Justify your answer. (10 points)
You are a budget analyst for the state of Confusion. The state legislature has decided to impose a per-unit output tax on the firms that supply gropnooks in the state of Confusion. The leader of the legislature has come to you for advice. She wants to know the value at which the per-unit output tax needs to be set in order to raise a total of $375 (per week) from this tax.

Your knowledge of the market for gropnooks in the state of Confusion is very limited. However, you do know the following:

- the market for gropnooks is perfectly competitive
- the current market price of gropnooks is $20
- 100 gropnooks are sold/consumed each week
- the own-price elasticity of the demand for gropnooks, evaluated at $P_G=20$, is -4 ($e_G = -4.0$)
- the own-price elasticity of the supply of gropnooks, evaluated at $P_G=20$, is 2 ($\gamma_G = 2.0$).

a. Use this information to derive linear functions for the market demand for gropnooks and the market supply of gropnooks. Demonstrate that your demand and supply functions are in fact consistent with this information. (HINT: Begin by “working backwards” from the elasticities.) (10 points)

b. At what value should the state government set the per-unit output tax if its goal is to raise a total of $375 (per week)? Show all your work. (20 points)

c. Demonstrate that your answer in part b is correct. (10 points)
3. The initial market demand for good X is given by

(1) \[ X_D = 2000 - 40P_X + 20P_Y \]

where \( X_D \) is the quantity of good X demanded, \( P_X \) is the market price of good X, and \( P_Y \) is the market price of good Y. The initial market supply of good X is given by

(2) \[ X_S = 20P_X \]

where \( X_S \) is the quantity of good X supplied. The initial market demand for good Y is given by

(3) \[ Y_D = 750 + 20P_X - 40P_Y \]

where \( Y_D \) is the quantity of good Y demanded. The initial market supply of good Y is given by

(4) \[ Y_S = 80P_Y \]

where \( Y_S \) is the quantity of good Y supplied.

a. Derive the initial equilibrium market price of good X, the initial equilibrium market quantity of good X, the initial equilibrium market price of good Y, and the initial equilibrium market quantity of good Y. Illustrate these market equilibria with LARGE, COMPLETELY LABELED diagrams (one diagram for the market for good X and another diagram for the market for good Y). (15 points)

Suppose good X is produced domestically and good Y is imported. In an attempt to assist the domestic producers of good X, the government imposes a 100 unit import quota on good Y (\( Y \leq Y^Q = 100 \)). In addition, the government sets a price floor of $60 in the market for good X (\( P^F_X = 60 \)), and agrees to purchase, at \( P^F_X = 60 \), all units of good X firms produce but do not sell in the market.

b. How much will the government spend on this combined quota/price support program? How will this program affect the market price of good X, the market quantity of good X, the market price of good Y, and the market quantity of good Y? Revise/edit the diagrams you drew for part a to illustrate these price and output effects. Be sure to identify/label the government’s total spending on the combined quota/price support program on your diagram(s). (20 points)
4. The market demand for good X in (time) period t is given by

\[ X_{D,t} = \alpha - \beta P_t \quad (\alpha > 0, \beta > 0) \]

where \( X_{D,t} \) is the quantity of good X demanded in period t and \( P_t \) is the market price of good X in period t. The market supply of good X in period t is given by

\[ X_{S,t} = bP_t^e \quad (b > 0) \]

where \( X_{S,t} \) is the quantity of good X supplied in period t and \( P_t^e \) represents the suppliers’ expectation of the market price of good X in period t. The suppliers’ expectation of the market price of good X in period t, as formed in period t, is given by

\[ P_t^e = \gamma P^* + (1 - \gamma) P_{t-1} \quad (0 < \gamma < 1) \]

where \( P_{t-1} \) is the market price of good X in period t-1 and \( P^* \) is the long-run equilibrium market price of good X.

The market price in period t adjusts such that the quantity of X demanded in period t is equal to the quantity of X supplied in period t. That is, market equilibrium in any period t is defined by

\[ X_{D,t}(P_t) = X_{S,t}(P_t). \]

A long-run equilibrium is reached when

\[ P^* = P_t = P_{t-1} = P_{t-2} = \ldots \]

a. Discuss how the suppliers make their output decisions in period t. Be sure to address what the suppliers probably would “like” to know, what they do know, and what they don’t know, given the supply function defined above in equation 2 and the price expectations function defined above in equation 3. (10 points)

b. Derive an equation that relates the market price of good X in period t (\( P_t \)) to the market price of good X in period t-1 (\( P_{t-1} \)) and the long-run equilibrium market price of good X (\( P^* \)). Explain why the relationship you derived “makes sense” within the context of this model. (15 points)

c. Derive the long-run equilibrium market price of good X (\( P^* \)) and the long-run equilibrium market quantity of good X (\( X^* \)). Is the suppliers’ price expectation “correct” when the market is in long-run equilibrium? Explain why this must be the case. (10 points)

THIS SECTION CONTINUES ON THE NEXT PAGE.
For this market to be stable in the long run, the market price must converge over time. That is, \( P_{t+1} \) must be “closer” to \( P_t \) than \( P_{t+2} \) is to \( P_{t+1} \); \( P_{t+2} \) must be “closer” to \( P_{t+1} \) than \( P_{t+3} \) is to \( P_{t+2} \); and so on. Mathematically, convergence requires

\[
|P_t - P_{t+1}| < |P_{t+1} - P_{t+2}| < |P_{t+2} - P_{t+3}| < \ldots \ldots
\]

d. Derive the condition under which the long-run equilibrium in this market will be stable. What does this condition mean in terms of the demand and supply functions defined above? Illustrate with a demand/supply diagram. What will happen in this market if this stability condition does not hold? Illustrate with a demand/supply diagram. (20 points)

e. Assume the long-run equilibrium in this market is stable. Now, consider this market in some arbitrarily selected period \( t \). Suppose \( P_t < P^* \). Will this market ever reach a long-run equilibrium in some future period \( \tau \) such that \( P_\tau = P^* \) in period \( \tau \) and in all subsequent periods? Explain. (15 points)