EXAM 2

This exam is worth 400 points. The point-value of each question is stated in parentheses after the question.

1. Each of two firms (F₁ and F₂) has one job opening. Suppose the firms offer different wages. Let \( w₁ \) represent the wage offered by F₁. Let \( w₂ \) represent the wage offered by F₂. Assume \( \frac{1}{2}w₁ < w₂ < 2w₁ \).

There are two workers (L₁ and L₂), each of whom can apply to one and only one of these firms. The workers simultaneously and independently decide whether to apply to F₁ or F₂. If only one worker applies to a given firm, then that worker gets the job at that firm. If both workers apply to the same firm, then the firm hires L₁ with probability 1/2 and hires L₂ with probability 1/2. For now, assume an unemployed workers gets a payoff of zero. Assume both workers have identical preferences over the wage they receive. For simplicity, assume \( U(w) = w \) for both workers. (NOTE: Both workers are risk neutral.)

   a. Set up the normal form representation of this game. (HINT: The payoffs when both workers apply to the same firm will be represented by expected wages.) (10 points)

   b. Identify the Nash equilibria in pure actions. Explain why these are in fact Nash equilibria in pure actions. (10 points)

   c. Find the Nash equilibrium in mixed strategies. Consider the mixed strategy of L₁. Under what condition(s) will this worker be more likely to apply to F₁ rather than F₂? Under what condition(s) will this worker be equally likely to apply to F₁ and F₂? Discuss the economic intuition underlying these results. (20 points)

In an effort to simplify the analysis somewhat, assume \( w₁ = 4 \) and \( w₂ = 6 \) for the rest of the questions in this section. (NOTE: DO NOT adopt this assumption for parts a, b, and c.)

   d. What is the probability that one of these workers will be unemployed? (10 points)

Suppose the government awards unemployment compensation to any worker who is unemployed. A worker who is unemployed receives a payment of \( c = 1 \) from the government.

   e. Set up the normal form representation of this game when \( w₁ = 4 \), \( w₂ = 6 \), and \( c = 1 \). (10 points)

   f. Suppose both workers play equilibrium mixed strategies. What is the probability that one of these workers will be unemployed? (15 points)

   g. Compare your answer to part d to your answer to part f. How does the availability of unemployment compensation affect the probability that one of the workers will be unemployed? Discuss the economic intuition underlying this result. (15 points)
2. Suppose there are two firms selling gropnooks. Let \( a_i \) represent the advertising level of firm \( i \) \((i=1,2)\). Assume the firms’ profits are define as follows:

\[
\begin{align*}
(1) \quad \pi_1(a_1,a_2) &= 4a_1 + 3a_1a_2 - a_1^2 \\
(2) \quad \pi_2(a_1,a_2) &= 2a_2 + a_1a_2 - a_2^2
\end{align*}
\]

where \( \pi_i(a_1,a_2) \) represents the profits of firm \( i \) \((i=1,2)\).

a. Derive firm 1’s best response function and firm 2’s best response function. What do these best response functions (strategies) suggest about the nature of the strategic interdependence between firm 1 and firm 2? \(10 \text{ points} \)

b. Suppose both firms choose their respective advertising levels simultaneously and independently. Find the Nash equilibrium advertising levels for this static game. Explain why these advertising levels do in fact constitute a Nash equilibrium in this game. Use numerical examples, derived within the context of this model, to illustrate your explanation. \( \text{(NOTE: A diagram with both best response functions might be helpful here.)} \) \(15 \text{ points} \)

3. Suppose there are three firms (firm 1, firm 2 and firm 3), each producing its own (differentiated) brand of good \( X \). Let \( X_i \) represent the output of firm \( i \) \((i=1,2,3)\). Let \( P_i \) represent the price charged by firm \( i \) \((i=1,2,3)\).

The demand and cost conditions facing firm 1 are given by

\[
\begin{align*}
(1) \quad X_1(P_1,P_2,P_3) &= 2000 - 2P_1 + P_2 + P_3 \\
(2) \quad C_1(X_1) &= 20X_1.
\end{align*}
\]

The demand and cost conditions facing firm 2 are given by

\[
\begin{align*}
(3) \quad X_2(P_1,P_2,P_3) &= 1700 + P_1 - 2P_2 + P_3 \\
(4) \quad C_2(X_2) &= 40X_2.
\end{align*}
\]

The demand and cost conditions facing firm 3 are given by

\[
\begin{align*}
(5) \quad X_3(P_1,P_2,P_3) &= 1400 + P_1 + P_2 - 2P_3 \\
(6) \quad C_3(X_3) &= 60X_3.
\end{align*}
\]

The firms know each other’s demand functions and cost functions. That is, there is complete information.

THIS SECTION CONTINUES ON THE NEXT PAGE.
a. State the profit function for each firm. (5 points)

b. Derive each firm’s best response function. (10 points)

c. Consider firm 1’s best response function. According to this best response function, how will firm 1 alter the price it charges in response to increases in the prices charged by firm 2 and firm 3? Discuss the economic intuition underlying firm 1’s response in the face of rising competitors’ prices. (10 points)

Suppose these firms choose their prices simultaneously and independently.

d. Derive the equilibrium prices, output levels, and profits. Explain why the prices you have identified here are in fact equilibrium prices. (20 points)

e. Calculate the “weighted average” price of good X. The weighted average price in this case can be defined as $\overline{P} = \theta_1 P_1 + \theta_2 P_2 + \theta_3 P_3$, where $\theta_i$ represents the market share of firm i (i=1,2,3). (10 points)

Now, suppose these firms choose their prices sequentially. Firm 1 publicly announces its price first. Firm 2 publicly announces its price second. Firm 3 publicly announces its price last. A firm may not change its price after its public announcement.

f. Derive the equilibrium prices, output levels, and profits. Justify the solution methodology you employ. Explain why the prices you have identified here are in fact equilibrium prices. (30 points)

g. Calculate the “weighted average” price of good X when prices in this market are set sequentially. (REMEMBER: Don’t forget to calculate the new market shares.) (10 points)

h. Explain how and why the firms in this market benefit from setting prices sequentially rather than simultaneously. Explain how and why the market shares differ under the two pricing schemes. (NOTE: It is not sufficient to compare prices, profits, and market shares under the two pricing schemes. You must provide an “intuitively appealing” explanation of why equilibrium prices, profits, and market shares differ under the two pricing schemes.) (15 points)

i. Are the buyers of good X better off under the sequential pricing scheme or the simultaneous pricing scheme? Discuss the economic intuition underlying your conclusion. (10 points)
4. The inverse market demand for good X is given by

\[ P(X) = 50 - 0.1X. \]

Currently, this market is served by a single firm (firm 1). Firm 1’s total cost of production is given by

\[ C_1(X_1) = 0.025X_1^2 \]

where \( X_1 \) represents firm 1’s output.

a. How much profit will firm 1 earn if it faces absolutely no chance of having to share the market with another firm? (5 points)

Now, suppose another firm (firm 2) is considering entering this market. Firm 2’s total cost of production is given by

\[ C_2(X_2) = 10X_2 + 0.025X_2^2 \]

where \( X_2 \) represents firm 2’s output.

b. Demonstrate that firm 2 is at a cost disadvantage relative to firm 1. (5 points)

c. How much profit does firm 2 expect to earn \textbf{IF} it believes firm 1 will hold its output constant at the level you identified in part a? Is this a reasonable belief on firm 2’s part? Explain. (10 points)

d. Firm 1 has learned about the possibility that firm 2 might enter the market. In response to this new information, firm 1 considers setting a level of output (\( \bar{X}_1 \)) such that firm 2 will not enter the market. What level of output would firm 1 have to commit to in order to deter firm 2 from entering the market? How much profit will firm 1 earn if it commits to this entry-deterring level of output? (HINT: You must solve for \( \bar{X}_1 \) such that firm 2’s best response to \( \bar{X}_1 \) generates zero profit for firm 2.) (15 points)

e. Suppose firm 1 and firm 2 instead play a Cournot quantity game if and when firm 2 decides to enter the market. What are firm 1’s and firm 2’s profits in this case? (10 points)

f. Is it reasonable to believe that firm 1 will try to commit to the \( \bar{X}_1 \) you identified in part d in order to deter entry? Explain. (10 points)
5. The inverse market demand for good X is given by

(1) \( P(X) = 100 - .1X \).

Currently, this market is served by a single firm (firm 1). Firm 1’s total cost of production is given by

(2) \( C_1(X_1) = 10X_1 + 9000 \)

where \( X_1 \) represents firm 1’s output and 9000 is firm 1’s sunk costs.

Firm 1 has learned that another firm (firm 2) might enter the market. Firm 2’s total cost of production is given by

(3) \( C_2(X_2) = 16X_2 + 6000 \)

where \( X_2 \) represents firm 2’s output and 6000 is the sunk cost firm 2 will incur if it does indeed enter the market.

a. What is the minimum level of output firm 1 would have to commit to (\( X_1 \)) in order to deter firm 2 from entering the market? (20 points)

b. Would you expect firm 1 to actually produce the minimum entry-deterring level of output you identified in part a? Explain. (10 points)
6. Sassy Sally (SS) and Trusty Rusty (TR) each own gas stations in Mt. Vermin. Each has three different pricing options: \( P_{SS} = P_{TR} = \{P_H, P_M, P_L\} = \{1.50, 1.25, 1.00\} \). The payoffs associated with all possible combinations of prices are given by the following payoff matrix, where the payoff pairs are \((\pi_{SS}, \pi_{TR})\).

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Suppose we model the relationship between Sally and Rusty within the context of a static game of complete information.

a. What is the solution to this game if the players are not allowed to communicate before they choose their respective prices? Justify your solution. (10 points)

b. Suppose we allow the players to communicate before they choose their prices. They may talk to each other about the game before they play the game. However, there is no possibility of any further interaction between the two players after they choose their respective prices. What is the solution to this game in this setting? Justify your solution. (10 points)

Now, suppose we model the relationship between Sally and Rusty within the context of an infinite repeated game. On any given round of the game, they choose their prices simultaneously and independently. Assume both players have perfect recall. That is, on any given round of the game, they know the complete history of the game up to that round.

Sassy Sally decides to adopt the following trigger strategy.

\[
P_{SS}(\tau) = \begin{cases} 
1.25 & \text{if } \tau = 1 \\ 
1.25 \text{if } \tau > 1 \text{ and } P_{TR}(t) = P_{SS}(t) = 1.25 \text{ for all } t = 1, \ldots, \tau - 1 \\ 
1.00 & \text{otherwise} 
\end{cases}
\]

c. Sally’s trigger strategy consists of a “promise” and a “threat.” Identify the threat. Is the threat credible? Explain why the credibility of Sally’s threat is an important factor in establishing and maintaining cooperative pricing behavior in this game. (10 points)
Rusty somehow has discovered Sally’s strategy before the first round of the game.

d. Suppose Rusty’s discount factor is equal to .1 (δTR=.1). Discuss, in detail, the pricing decision Rusty faces on the first round of the game, given Sally’s trigger strategy and Rusty’s discount factor. (NOTE: I expect to see some present value calculations here.) Would you expect to see cooperative pricing behavior in this setting? Explain. (15 points)

e. Suppose Rusty’s discount factor is equal to .8 (δTR=.8). Discuss, in detail, the pricing decision Rusty faces on the first round of the game, given Sally’s trigger strategy and Rusty’s discount factor. (NOTE: I expect to see some present value calculations here.) Would you expect to see cooperative pricing behavior in this setting? Explain. (15 points)

f. Suppose Rusty does in fact decide to cooperate with Sally. Would it make sense for Rusty to adopt the same trigger strategy employed by Sally? Explain. (10 points)

g. Explain why cooperative pricing behavior is possible in the infinite repeated version of this game but is not at all likely in the static version of this game. (10 points)